EFFECT OF A PROTECTIVE COATING ON THE FREQUENCY CHARACTERISTICS OF FILM TRANSDUCERS IN THERMOANEMOMETERS

I. L. Povkh and D. F. Shtopko

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An expression is derived for the thermal characteristic impedance. Equations are obtained for calculating the amplitude-frequency and the phase-frequency characteristics of planar-film transducers with protective coating.

Because of their excellent dynamic characteristics, film transducers are successfully used in thermoanemometers for measuring all kinds of flow parameters in turbulent fluids. In certain measurements (turbulence intensity, transverse correlations) it is necessary to know the amplitude-frequency and the phase-frequency characteristics. Furthermore, these characteristics are also needed for the design of feedback amplifiers for thermoanemometers.

Direct and indirect methods of measuring the frequency characteristics of film transducers as well as the underlying theoretical principles have been outlined in [1-4]. It has been shown that the high-frequency ranges of their characteristics are identical for film transducers of various shapes. The low-frequency characteristics depend considerably on the transducer geometry.

We will analyze here the one-dimensional version of the problem concerning the effect of protective coating on the frequency characteristics of a planar-film transducer.

For a precise compensation of the film inertia it is necessary to know the dynamic characteristic of the sensing element in the transducer. The most direct method of analysis is to plot the transfer function, which characterizes a linear system in terms of a simple harmonic parameter.

The thermal model and the electrical equivalent of such a transducer are shown in Fig. 1.

According to Fig. 1b, the equivalent electric circuit consists of a resistance and impedances. Generally, Z_T' and Z_T'' are analogs of RC-networks. The transfer function can then be written as

$$W(j\omega)=\frac{\zeta_1 j\omega+1}{\zeta_2 j\omega+1}, \tag{1}$$

with ζ_1 and ζ_2 denoting the closed-loop and the open-loop time constant respectively, and where $j = \sqrt{-1}$.

In the transducer–thermoanemometer system ζ_1 would be the time constant of the amplifier and ζ_2 would be the time constant which characterizes the thermal inertia of the transducer.

For an open system $\zeta_1 = 0$ and Eq. (1) simplifies to

$$|W(j\omega)|=1/\sqrt{1+(\omega\zeta_2)^2}. \tag{2}$$

This equation is identical, except for a constant, to the well known formula [5] for calculating the amplitude-frequency characteristics of thermoanemometer transducers.

In order to account for the effect of a dielectric coating over the transducer film on the frequency characteristics of such a transducer, it is necessary either to measure the time constant of thermal inertia or to determine the transfer function in terms of the given transducer parameters.

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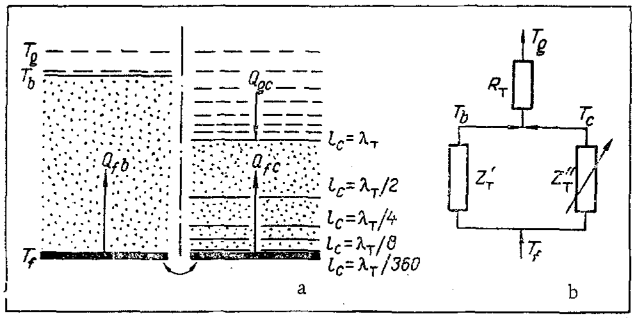

Fig. 1. (a) Thermal model and (b) the electrical equivalent of a planar-film transducer with protective coating, for a thermoanemometer.

Let T_f be the temperature of the hot film. The thermal fluxes from film to substrate and from film to coating will then be

$$Q_{fb}=k_1\int\limits_s \frac{\partial T}{\partial x}\bigg|_{x=0} dS;\ Q_{fc}=k_2\int\limits_s \frac{\partial T}{\partial x}\bigg|_{x=0} dS. \tag{3}$$

Here k_1 and k_2 denote the thermal conductivity of the substrate material and of the coating material respectively.

The quantities of heat transferred from a unit area of substrate surface and of coating surface per unit time are respectively

$$Q_{gb}=\alpha(T_g-T_b);\ Q_{gc}=\alpha(T_g-T_c), \tag{4}$$

with T_b, T_c, and T_g denoting the temperature of the substrate, of the coating, and of the ambient fluid respectively.

We will examine the heating of the film as a harmonic process. In this case

$$T_f=T_0\cos\omega t. \tag{5}$$

Under stable equilibrium conditions in the solid state, the temperature distribution produced by harmonic fluctuations of the surface temperature can be expressed as [6]:

$$T_x=T_0\exp\left(-\frac{2\pi x}{\lambda_T}\right)\cos\left(\omega t-\frac{2\pi x}{\lambda_T}\right). \tag{6}$$

Here $\lambda_T = 2\pi/\sqrt{\omega/2a}$.

Differentiating Eq. (6) and inserting appropriate values into (3), we obtain

$$Q_{fb}|_{x=0}=-ldk_1\sqrt{\frac{\omega}{2a_1}}\,T_0(\sin\omega t+\cos\omega t); \tag{7}$$

$$Q_{fc}|_{x=0}=-ldk_2\sqrt{\frac{\omega}{2a_2}}\,T_0(\sin\omega t+\cos\omega t); \tag{8}$$

$$Q_{fc}|_{x=l_c}=ldk_2\sqrt{\frac{\omega}{2a_2}}\,T_0\exp\left(-\frac{2\pi l_c}{\lambda_T}\right)\times\left[\sin\left(\omega t-\frac{2\pi l_c}{\lambda_T}\right)-\cos\left(\omega t-\frac{2\pi l_c}{\lambda_T}\right)\right]. \tag{9}$$

Here a_1 and a_2 denote the thermal diffusivity of the substrate material and of the protective coating material respectively.

We will consider the case where the substrate and the coating are made of the same material such as, for example, Pyrex glass. Then $k_1 = k_2 = k$ and $a_1 = a_2 = a$. In this material at a thermal frequency $f = 1$ Hz the thermal wavelength is $\lambda_T = 2\cdot 10^{-3}$ m. From the viewpoint of the resulting temperature distribution it does not matter whether the substrate depth is infinite or equal to one thermal wavelength. With this in mind, we will assume that no heat is transferred to the stream of fluid from the substrate. Heat is transferred to the stream from the surface of the protective coating only.

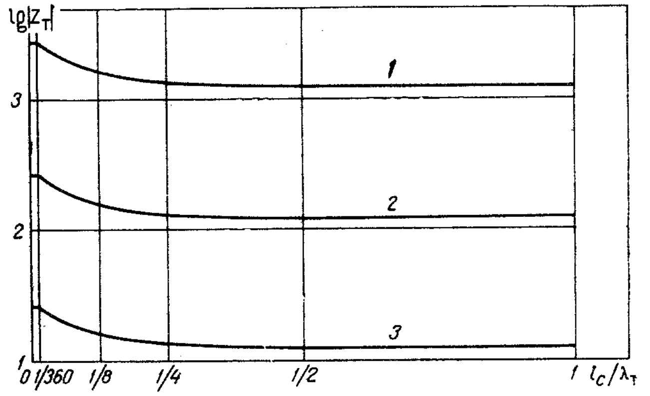

Fig. 2. Thermal characteristic impedance as a function of the dimensionless coating thickness at various frequencies: 1) f = 1 Hz; 2) 10^2 Hz; 3) 10^4 Hz.

If T_{l_c} denotes the temperature of the coating surface, then the quantity of heat transferred from that surface to the stream can be expressed by the following empirical formula [2]:

$$Q_{gc} = -dk_g \, \mathrm{Nu} \, T_{l_c}, \tag{10}$$

where Nu = $(0.519\,\mathrm{Re}^{0.5} - 0.04)\,\mathrm{Pr}^{0.3}$, Re = $\sqrt{ud/\nu_g}$, Pr = ν_g/a_g, u is the mean velocity of the stream, a_g is the thermal diffusivity of the fluid, and k_g is the thermal conductivity of the fluid.

The transient heat transfer from the film to the substrate and to the coating is determined by the thermal characteristic impedance Z_T (Fig. 1b). According to Fig. 1b, $Z_T = Z_T' + Z_T''$ and

$$Z_T = T_f / \left(2Q_{fb}|_{x=0} - Q_{fc}|_{x=lc}\right). \tag{11}$$

For simplification, we let $l_c = \lambda_T/360 \simeq 5\ \mu$. Then

$$Q_{fc}|_{lc=\lambda_T/360} = -ldk\sqrt{\frac{\omega}{2a}}\, T_0 \exp\left(-\frac{\pi}{180}\right)(\sin\omega t + \cos\omega t). \tag{12}$$

Inserting (7) and (12) into (11), we obtain

$$Z_T = 1/ldk\sqrt{\frac{\omega}{2a}}\left[2 - \exp\left(-\frac{\pi}{180}\right)\right](1+j). \tag{13}$$

The modulus of the thermal characteristic impedance $|Z_T|$ is shown in Fig. 2 as a function of the coating thickness at various frequencies. As the thermal frequency increases, the absolute value of Z_T decreases. This affects the sensitivity of the film transducer: the sensitivity decreases rapidly with increasing frequency. This means in physical terms that, as the thermal frequency increases, the thermal wavelength decreases and heat from the film is absorbed at low depths in the substrate and in the coating.

The thermal characteristic resistance R_T will be expressed as

$$R_T = (T_g - T_{l_c})/Q_{gc}. \tag{14}$$

The transfer function for a planar-film transducer with protective coating will be sought in the general form

$$W(j\omega) = Z_T/(Z_T + R_T). \tag{15}$$

Solving Eq. (15), we find

$$|W(j\omega)| = A \Big/ \sqrt{(A + B\sqrt{f})^2 + A^2}, \tag{16}$$

where

$$A = dk_g\,\mathrm{Nu}\exp\left(-\frac{2\pi l_c}{\lambda_T}\right);\quad B = dlk\sqrt{\frac{\omega}{2a}}\left[2 - \exp\left(-\frac{2\pi l_c}{\lambda_T}\right)\right].$$

The temperature phase shift will be defined as

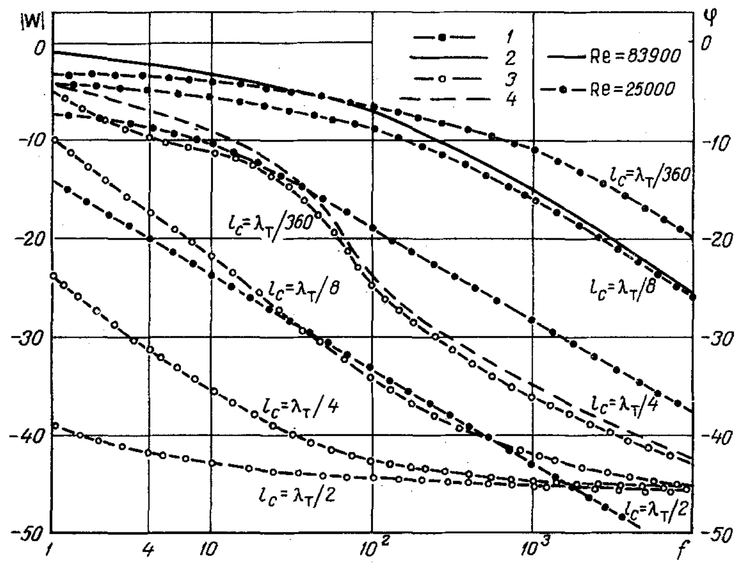

Fig. 3. Amplitude-frequency and phase-frequency characteristics of planar-film transducers: 1) amplitude-phase characteristics of transducers with various thicknesses of protective coating; 2) amplitude-frequency characteristic of a transducer without coating (experiment in [3]); 3) phase-frequency characteristics of transducers with various thicknesses of protective coating; 4) experimental phase-frequency characteristic of a transducer without coating [1]. Phase shift φ (deg), frequency f (Hz).

$$\operatorname{tg}\varphi = -\frac{dlk\sqrt{\frac{\omega}{2a}}\left[2-\exp\left(-\frac{2\pi l_c}{\lambda_\text{т}}\right)\right]}{dlk\sqrt{\frac{\omega}{2a}}\left[2-\exp\left(-\frac{2\pi l_c}{\lambda_\text{т}}\right)\right]+dk_g\,\text{Nu}\exp\left(-\frac{2\pi l_c}{\lambda_\text{т}}\right)}. \tag{17}$$

Amplitude-frequency and amplitude-phase characteristics for a planar-film transducer with protective coating are shown in Fig. 3. The material of both substrate and coating here is Pyrex glass. It is easy to see that, as the coating thickness increases, the transducer sensitivity decreases rapidly. The temperature phase shift at high frequencies approaches 45°. The same is observed also in the case of uncoated film transducers.

For comparison, in Fig. 3 are shown also amplitude-frequency and phase-frequency characteristics of hot-film transducers according to [1] and [3]. A comparison between calculated characteristics of transducers with and without coating respectively indicates that a very thin coating ($\sim 5\ \mu$) has no appreciable effect on the transducer sensitivity.

All these conclusions are based on the assumption that steady-state heat transfer occurs from the surface of a hot film to the substrate and to the coating, and from there to the fluid stream. Heat losses in the film due to radiation are very small. Furthermore, any heat transfer to the end surfaces of the transducer has been disregarded here.

In view of this, the value obtained for the thermal low-frequency impedance is somewhat on the high side. A comparison between the amplitude-frequency characteristics of transducers with and without coating (experiment) justifies the assertion that this error is within limits acceptable in thermoanemometric measurements.

The results which have been shown here apply to a homogeneous coating–substrate system. Such transducers are easily made of quartz and various grades of glass. When the coating material is different from the substrate material, then formulas (16) and (17) can, in a somewhat more complex form, be used for any substrate–coating system.

NOTATION

ζ is the time constant;
ω is the radian frequency;
f is the frequency;
T is the temperature;
Q is the thermal flux;
k is the thermal conductivity;
α is the heat transfer coefficient;
a is the thermal diffusivity in solid state;
λ_T is the thermal wavelength;
l is the length of film on sensing element, normal to the direction of the stream;
d is the width of film on sensing element, parallel to the direction of the stream;
l_b is the thickness of substrate;
ν_g is the kinematic viscosity of fluid;
Z_T is the thermal characteristic impedance;
R_T is the thermal characteristic resistance;
φ is the phase-shift angle;
T_j is the temperature of hot film (°C);
T_c is the temperature of protective coating, at distance l_c from the film;
T_b is the temperature of substrate, at distance l_b from the film;
$l_c = \lambda_T$, $l_c = \lambda_T/2$, etc. are the thickness of protective coating, equal to thermal wavelength, equal to half the thermal wavelength, etc.;
Q_{fc} is the thermal flux from hot film to protective coating;
Q_{fb} is the thermal flux from hot film to substrate;
Q_{gc} is the heat transferred from protective coating to fluid stream;
Z'' is the characteristic impedance of transient heat transfer from film to coating (°C·sec/J);
Z' is the characteristic impedance of transient heat transfer from film to substrate (°C·sec/J);
W is the relative sensitivity, dB.

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